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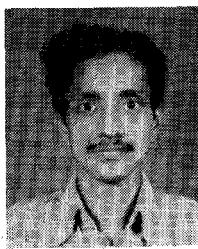


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## Calibration of Multiport Reflectometers by Means of Four Open/Short Circuits

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**Abstract**—This paper presents a simple method for calibrating any practical multiport reflectometer by means of four reflection standards with known complex reflection coefficients. It is shown that these four standards can be such that their reflection coefficient modulus = 1. Computer simulation proves that no singularity appears for both ideal and nonideal five- and six-port reflectometer in a wide range of phase distribution of reflection coefficients. A group of calibration results for a practical simple six-port is listed to show this calibration procedure; by the use of these calibrated network parameters, some measurement results are presented and compared with the values obtained at the National Bureau of Standard, U.S.A.

Both computer simulation and experimental results show that the

numerical singularities which may be encountered in multiport calibration procedures are not an intrinsic properties of multiport but from related mathematical treatment.

### I. INTRODUCTION

IT IS WELL KNOWN that the key problem for a network analyzer is its calibration. The existing self-calibration procedures for the six-port reflectometer [1], [3] can provide accurate results but are complex and cannot be directly used to calibrate the five-port. Another way to calibrate a network analyzer is via some reflection standards, which would be very useful for microwave engineering application. Woods [4] has discussed this problem in detail and concludes that seven standards are needed, of which at most five may have  $|\Gamma|=1$ , to avoid numerical

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singularities encountered in the calibration procedure.

The authors find that the singularities are related to mathematical treatment in the calibration procedure and are not the properties of the multiport reflectometer itself. For calibrating a practical multiport (with the number of ports  $N \geq 5$ ), the method presented in this paper requires a set of only four standards instead of the seven suggested by Woods. Furthermore, all these four standards can be such that their reflection coefficient modulus = 1; this is very easily achieved in practice and no singularities will be encountered in this procedure.

## II. BASIC EQUATION OF $N$ -PORT

For an arbitrary linear  $N$ -port microwave reflectometer (Fig. 1), the basic relationships between its network parameters and power readings can be expressed as follows:

$$P_i = \frac{P_{i+3}}{P_3} = q_i \left| \frac{1 + A_i \Gamma}{1 + A_0 \Gamma} \right|^2 \quad (1)$$

where  $P_3$  to  $P_N$  are  $N - 3$  power readings measured at port 3 to  $N$ , respectively;  $q_1$  to  $q_{N-3}$  are  $N - 3$  scalar parameters;

$$A_0 = \alpha_0 e^{j\phi_0} = a_0 + jb_0; \quad (2-1)$$

$$A_i = \alpha_i e^{j\phi_i} = a_i + jb_i; \quad (2-2)$$

$i = 1, 2, \dots, N - 3$ ; and

$$\Gamma = |\Gamma| e^{j\psi} = X + jY \quad (3)$$

is the complex reflection coefficient to be tested.

Obviously, the reflection coefficient  $\Gamma$  is one of the two intersections of two circles in the  $\Gamma$ -plane for a five-port [4]–[6]. When  $N = 6$ , the solution of (1) has a much simpler form as follows:

$$X = \frac{U_0 + U_1 p_1 + U_2 p_2 + U_3 p_3}{1 + C_1 p_1 + C_2 p_2 + C_3 p_3} \quad (4-1)$$

$$Y = \frac{V_0 + V_1 p_1 + V_2 p_2 + V_3 p_3}{1 + C_1 p_1 + C_2 p_2 + C_3 p_3}. \quad (4-2)$$

The relationship between the network parameters is (1), (2), and (4) can be found in the Appendix.

When  $N > 6$ , generally speaking, more port(s) are used to improve the properties of a six-port in a wide frequency band; the operation principle is still that of the six-port at any specified frequency in the band. Therefore, the measurement and calibration problems of a  $N$ -port ( $N > 6$ ) can be simplified in a set of that of six-ports.

To calibrate a six-port reflectometer, one possible way is from (4). This is the method suggested by Woods and discussed in detail in the literature [4].

From another point of view, one can calibrate the  $N$ -port reflectometer directly from (1). When a standard load with known reflection coefficient  $\Gamma_k$  is connected at port 2 of a  $N$ -port, the following  $N - 3$  equations can be found:

$$|\Gamma_k|^2 (q_i a_i^2 - p_i a_0^2) + 2X_k (q_i a_i - p_i a_0) + 2Y_k (p_i b_0 - q_i b_i) + q_i = p_i \quad (5)$$

where  $i = 1, 2, \dots, N - 3$ .

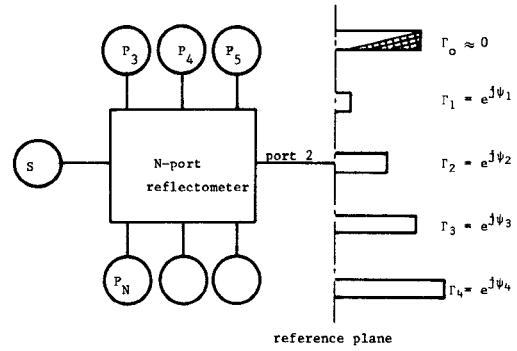


Fig. 1. Calibration procedure of  $N$ -port reflectometer by means of four standards with their reflection coefficient modulus = 1.

To solve these  $N - 3$  simultaneous equations, two methods can be used. One is taking the terms of  $q_i a_i^2$ ,  $a_0^2$ ,  $q_i a_i$ , and  $q_i b_i$  as independent variables. Then seven standards are needed and the same numerical singularities as in the literature [4] will be met. Another way is directly treating these  $N - 3$  simultaneous nonlinear equations obtained in a calibration measurement corresponding to a standard. To calibrate  $3N - 7$  unknown parameters,  $3N - 7$  equations are needed; therefore the minimum number of standards  $K$  are

$$K \geq \frac{3N - 7}{N - 3}. \quad (6)$$

When  $N = 5$  or 6,  $K = 4$ . Because of the nonlinearity of the equations, one must solve the ambiguous problems in the choice of roots, and the possible singularities must be examined.

## III. CALIBRATION METHOD

If a group of loads with reflection coefficient modulus = 1 but different in phase  $\psi_k$  are used as the calibration standards (Fig. 1), (5) becomes

$$q_i [1 + \alpha_i^2 + 2\alpha_i \cos(\psi_k + \phi_i)] = p_{i,k} [1 + \alpha_0^2 + 2\alpha_0 \cos(\psi_k + \phi_0)] \quad (7)$$

where  $k = 1, 2, 3$ , and 4, and  $i = 1, 2, \dots, N - 3$ .

Eliminating  $q_i$ ,  $\alpha_i$ , and  $\phi_i$  from (7), one finds the following results where  $j \neq k$  and  $j, k$  can be any two of the integers 1 to  $N - 3$ :

$$\alpha_0 = K_{j,k} - \sqrt{K_{j,k}^2 - 1} \quad (8)$$

$$\alpha_0 = \frac{E_k H_j - E_j H_k}{G_j H_k - G_k H_j} \cdot \frac{1 + \alpha_0^2}{2} \quad (9-1)$$

$$b_0 = \frac{E_k G_j - E_j G_k}{G_j H_k - G_k H_j} \cdot \frac{1 + \alpha_0^2}{2} \quad (9-2)$$

where

$$K_{j,k} = \frac{|G_j H_k - G_k H_j|}{\sqrt{(E_j H_k - E_k H_j)^2 + (E_k G_j - E_j G_k)^2}} \quad (10)$$

and

$$G_j = \sum_{i=1}^4 p_{j,i} \beta_i \cos \psi_i \quad (11-1)$$

$$H_j = \sum_{i=1}^4 p_{j,i} \beta_i \sin \psi_i \quad (11-2)$$

$$E_j = \sum_{i=1}^4 p_{j,i} \beta_i \quad (11-3)$$

$$\beta_1 = \sin(\psi_2 - \psi_3) + \sin(\psi_3 - \psi_4) + \sin(\psi_4 - \psi_2) \quad (12-1)$$

$$\beta_2 = \sin(\psi_3 - \psi_1) + \sin(\psi_1 - \psi_4) + \sin(\psi_4 - \psi_3) \quad (12-2)$$

$$\beta_3 = \sin(\psi_1 - \psi_2) + \sin(\psi_2 - \psi_4) + \sin(\psi_4 - \psi_1) \quad (12-3)$$

$$\beta_4 = \sin(\psi_3 - \psi_2) + \sin(\psi_1 - \psi_3) + \sin(\psi_2 - \psi_1). \quad (12-4)$$

The sign of the square root in (8) must be chosen as negative because any practical  $N$ -port has its  $\alpha_0 < 1$  (or, in other words,  $P_3 \neq 0$  in all possible measurements when a passive load is used). Substituting (8) and (9) into (7), it can be found

$$q_i = \xi_i \pm \sqrt{\xi_i^2 - \xi_i^2 - \eta_i^2} \quad (13)$$

$$a_i = \xi_i / q_i \quad (14-1)$$

$$b_i = \eta_i / q_i \quad (14-2)$$

where

$$\begin{aligned} \xi_i = \frac{1}{\beta_4} \left\{ \frac{1+\alpha_0^2}{2} [ p_{i,1} \sin(\psi_3 - \psi_2) \right. \\ \left. + p_{i,2} \sin(\psi_1 - \psi_3) + p_{i,3} \sin(\psi_2 - \psi_1) ] \right. \\ \left. + \alpha_0 [ p_{i,1} \cos(\phi_0 + \psi_1) \sin(\psi_3 - \psi_2) \right. \\ \left. + p_{i,2} \cos(\phi_0 + \psi_2) \sin(\psi_1 - \psi_3) \right. \\ \left. + p_{i,3} \cos(\phi_0 + \psi_3) \sin(\psi_2 - \psi_1) ] \right\} \quad (15-1) \end{aligned}$$

$$\begin{aligned} \xi_i = \frac{1}{\beta_4} \left\{ \frac{1+\alpha_0^2}{2} [ p_{i,1} (\sin \psi_2 - \sin \psi_3) \right. \\ \left. + p_{i,2} (\sin \psi_3 - \sin \psi_1) + p_{i,3} (\sin \psi_1 - \sin \psi_2) ] \right. \\ \left. + \alpha_0 [ p_{i,1} \cos(\phi_0 + \psi_1) (\sin \psi_2 - \sin \psi_3) \right. \\ \left. + p_{i,2} \cos(\phi_0 + \psi_2) (\sin \psi_3 - \sin \psi_1) \right. \\ \left. + p_{i,3} \cos(\phi_0 + \psi_3) (\sin \psi_1 - \sin \psi_2) ] \right\} \quad (15-2) \end{aligned}$$

and

$$\begin{aligned} \eta_i = \frac{1}{\beta_4} \left\{ \frac{1+\alpha_0^2}{2} [ p_{i,1} (\cos \psi_2 - \cos \psi_3) \right. \\ \left. + p_{i,2} (\cos \psi_3 - \cos \psi_1) + p_{i,3} (\cos \psi_1 - \cos \psi_2) ] \right. \\ \left. + \alpha_0 [ p_{i,1} \cos(\phi_0 + \psi_1) (\cos \psi_2 - \cos \psi_3) \right. \\ \left. + p_{i,2} \cos(\phi_0 + \psi_2) (\cos \psi_3 - \cos \psi_1) \right. \\ \left. + p_{i,3} \cos(\phi_0 + \psi_3) (\cos \psi_1 - \cos \psi_2) ] \right\} \quad (15-3) \end{aligned}$$

where  $i = 1, 2, \dots, N-3$ .

To determine the sign of the square root in (13), an additional measurement must be made in which an ordinary matched load is connected at port 2. From (5), the measured powers  $p_{i,0} \approx q_i$  (with an error less than  $\pm 5$  percent, depends on the quality of the load). This approximate value can be used to compare the two roots of (13), the true sign is then easily found, and the original point in the  $\Gamma$ -plane is then recognized. Because this matched load is not used as a standard, its error in reflection coefficient will not effect the calibration results. At this point, all network parameters are evaluated.

#### IV. COMPUTER SIMULATION

In order to prove the above method and to examine the possible singularities, five modules are established and simulated with the computer. As shown in Table I, there are two five-ports—an ideal five-port suggested by Riblet [5]; and a nonideal one—and three six-ports—an ideal and two nonideal ones. The standards used to calibrate them are shown in Fig. 2; all standards are such that its reflection coefficient modulus = 1.

Three categories in Fig. 2, having a different phase distribution, are respectively related to the following practical cases: four shorts with linear independent lengths, four short positions with equivalent distances, and a fixed short/open plus a known length of precision transmission line.

After the simulation at any possible combination of Table I and Fig. 2, we find the following to be true.

1) No singularity can be found except when  $\beta_4$  (equation (12-4)) equals zero. For a six-port, the three groups of possible solutions in (8) and (9) are always exactly the same.

2) The minimum phase difference between any two standards is preferably greater than  $\pi/4$ . If not, a higher calculating accuracy will be required, or in other words, higher power measurement accuracy will be required. From this point of view, the optimum phase distribution of standards (in Fig. 2(b) and (c)) is  $\Delta\psi = \pi/2$ .

3) The two possible solutions of (13) may be close to each other for a nonideal five- and six-port, and any mistake in the choice of  $q_i$  will give the wrong calibration

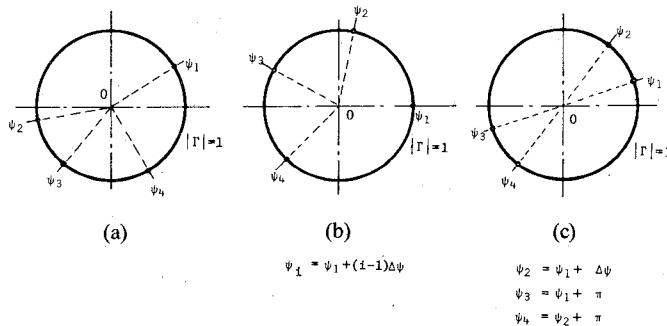


Fig. 2. Phase distribution of reflection standards of  $|\Gamma|=1$  for calibrating  $N$ -port reflectometer.

TABLE I  
THE  $N$ -PORT MODULES USED IN COMPUTER SIMULATION

Network Parameters	Five-port		Six-port		
	ideal	non-ideal	ideal	non-ideal 1	non-ideal 2
$A_0$	0	$.17 e^{j \cdot 22\pi}$	0	$.08 e^{j \cdot 31\pi}$	$.21 e^{j \cdot 143\pi}$
$A_1$	1	$.84 e^{j \cdot 472\pi}$	$0.8 e^{j \frac{\pi}{6}}$	$.56 e^{j \cdot 11\pi}$	$.76 e^{j \cdot 155\pi}$
$A_2$	$1 e^{j \frac{\pi}{2}}$	$.76 e^{-j \cdot 92\pi}$	$0.8 e^{j \frac{5\pi}{6}}$	$1.06 e^{j \cdot 532\pi}$	$.82 e^{j \cdot 751\pi}$
$A_3$	-	-	$0.8 e^{-j \frac{\pi}{2}}$	$.89 e^{j \cdot 945\pi}$	$.93 e^{-j \cdot 443\pi}$

which can easily be found from discontinuities in the frequency response or from measurements on a known value. To avoid this mistake, the reference matched load must be that of  $VSWR \leq 1.05$ .

## V. EXPERIMENTAL RESULTS

Typical experimental results are shown in Table II at a frequency of 3.4 GHz. The six-port under test is composed of a directional coupler plus three probes (Fig. 3); microwave power is measured by Schottky diode detectors of a PMI-1045 power meter, and all the system is controlled by a single board AIM-65 microcomputer via IEEE-488 bus.

In section "A" of Table II, the four reflection coefficients (real and imaginary) of related standards are that of four short positions separated every  $\lambda/8$  of a precision sliding short (or  $\Delta\psi = \pi/2$  in Fig. 2(b)). With these four standards, four groups of power reading can be obtained as listed in columns 1 through 4 in section "B". From the known reflection coefficients and related power readings, we then evaluate network parameters of the six-port by the use of the method reported (from (8) to (15)). In this procedure, two groups of intermediate results are listed in part "C" of Table II. One group offers three possible solutions for  $\alpha_0 e^{j\phi_0}$  (sequentially taking  $j, k$  as 1, 2; 2, 3; and 3, 1, respectively, see (8) through (11)); the small differences observed are from both the error of the standards and of power measurements. Another group is two possible roots of  $q_i$  (see (13)). In order to choose the correct root from the two roots, a reference measurement is taken under a matched load. In this case, the measured power readings are listed in the 5th column in part "B" of Table II. A comparison of the two roots of  $q_i$  with the 5th column

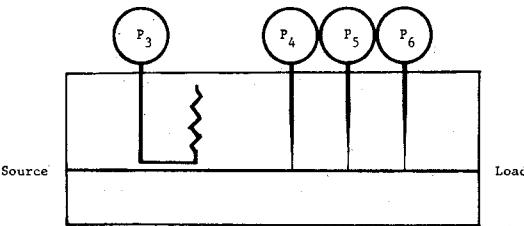


Fig. 3. An experimental six-port reflectometer working in the frequency band of 2-4 GHz.

TABLE II  
ONE PRINTED SHEET CALIBRATING A SIX-PORT REFLECTOMETER  
BY MEANS OF A PRECISION SLIDING SHORT

CALIBRATION OF SIX-PORT									
CALIBRATION FREQUENCY = 3.4 00GHZ									
A	OUR STANDARD	X	Y						
	1	1.0000	0.0000						
	2	0.0000	1.0000						
	3	-1.0000	0.0000						
	4	0.0000	-1.0000						
POWER READINGS 1 2 3 4 5 6 7									
P1/P3 0.7444 9.1283 23.9332 14.2561 7.3451 12.7933 5.9749									
P5/P3 13.5115 4.4361 0.4966 16.5196 5.0234 2.8343 13.5115									
P6/P3 7.5336 14.1251 12.7350 0.2742 5.9256 15.4156 1.4932									
THREE POSSIBLE ALPHA 0 PHI 0									
1 0.2141 -1.0742									
2 0.2120 -1.0473									
3 0.2112 -1.0712									
TWO SOLUTIONS OF Q 1 7.3706 3.1493									
TWO SOLUTIONS OF Q 2 4.8837 3.8223									
TWO SOLUTIONS OF Q 3 5.8537 4.1981									
CALIBRATED RESULTS A B Q U V C									
1 0.1030 -0.1857 7.3706 0.2267 0.1372 0.0294									
2 -0.6569 -0.0656 7.8837 -0.0651 -0.0516 0.0064									
3 0.8571 0.2525 5.8537 0.0420 -0.0562 -0.0384									
4 -0.0559 -0.8535 0.0082 0.0082 0.0084									
MEASURED VALUE MOD REF PHASE REF									
WILTRON 28A50-1 0.0086 0.005 0.2972 -2.903									
SHORT 22A 0.9997 1.005 2.0857 3.2153									
OPEN 22A 1.0088 1.000 -1.0408 .0339									

of power readings allows the network parameters of the six-port to be obtained and listed in part "D" of Table II in two forms:  $a_i + jb_i, q_i$  (equations (1) and (2)) or  $C_i, V_i$ , and  $U_i$  (equation (4)).

To examine the reliability of these calibration results, three reflection coefficients from a Wiltron matched load (type 28A50-1), and a fixed short/open (type 22A) are measured, and the related power readings are given respectively in the 5th through 7th column, part "B" of Table II. The measured complex reflection coefficient is given in part "E" of Table II, and compared with their reference data taken at the National Bureau of Standards in Boulder, CO. It must be noted that the measured phases are difficult to compare to each other, because of their different reference plane; however, the difference between the phases of the short and open circuits is close to  $\pi$  in both the reference and measured values as expected. Another example is a group of measured reflection coefficients from a

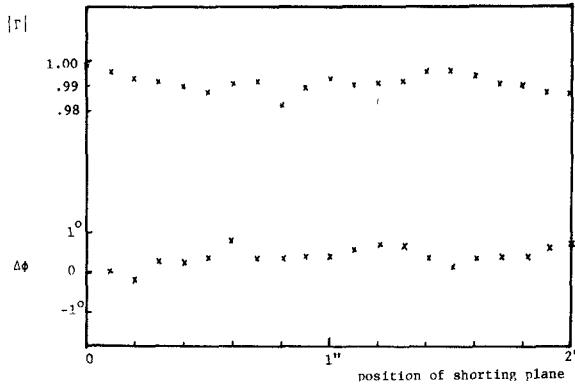


Fig. 4. Measured reflection coefficient of a precision Narda 901 NF sliding short at 4 GHz.

precision Narda 901NF-type sliding short as shown in Fig. 4. It shows that, by means of the above calibrated six-port reflectometer, the measured  $|\Gamma| = 0.99 \pm 0.01$  and  $|\Delta\phi| \leq 1^\circ$  in all 2 in movable range, where  $\Delta\phi$  means the phase differences between measured phase shift and the phase shift calculated from mechanical distance of shorting plane. More experimental results, including some intermediate values of reflection coefficient, for a five-port reflectometer can be found in the literature [6].

## VI. CONCLUSION

For calibrating an  $N$ -port reflectometer by means of reflection standards with known complex reflection coefficient, there are two problems be encountered: singularities and ambiguities. To avoid the ambiguities, one can start from (4) or (5) (taking all higher order terms as independent variables). But then numerical singularities may be met; therefore, the seven reflection standards must be chosen carefully as pointed out by Woods [4]. A much simpler method reported in this paper shows that no singularity will be encountered and the ambiguities can be solved by an analytical method (for  $\alpha_0$ ) or by a comparison method (for  $q_i$ ) by starting from the nonlinear equation (7). Therefore, the singularities in  $N$ -port calibration are merely from related mathematical treatment and are not the intrinsic properties of an  $N$ -port.

By the use of the reported simple calibration method, four reflection standards having  $|\Gamma| = 1$  are required. The kind of standards required are the most accurate, reliable, and easily achieved, and, therefore, the calibration errors from the standards are greatly decreased. This feature makes the reported method especially suitable to calibrate a practical engineering five- or six-port reflectometer based on crystal diode detectors.

## APPENDIX

The relationships between the network parameters in (1) and (4) for a six-port are

$$C_1 = \frac{q_2 q_3}{\Delta} [\alpha_0^2 (a_3 b_2 - a_2 b_3) + \alpha_2^2 (a_0 b_3 - b_0 a_3) + \alpha_3^2 (a_2 b_0 - b_2 a_0)]$$

and

$$C_2 = \frac{q_1 q_3}{\Delta} [\alpha_0^2 (a_1 b_3 - a_3 b_1) + \alpha_1^2 (a_3 b_0 - a_0 b_3) + \alpha_3^2 (a_0 b_1 - a_1 b_0)]$$

$$C_3 = \frac{q_1 q_2}{\Delta} [\alpha_0 (a_2 b_1 - a_1 b_2) + \alpha_1^2 (a_0 b_2 - a_2 b_0) + \alpha_2^2 (a_1 b_0 - a_0 b_1)]$$

$$U_0 = \frac{q_1 q_2 q_3}{2 \Delta} [\alpha_1^2 (b_2 - b_3) + \alpha_2^2 (b_3 - b_1) + \alpha_3^2 (b_1 - b_2)]$$

$$U_1 = \frac{q_2 q_3}{2} [\alpha_0^2 (b_3 - b_2) + \alpha_2^2 (b_0 - b_3) + \alpha_3^2 (b_2 - b_0)]$$

$$U_2 = \frac{q_1 q_3}{2 \Delta} [\alpha_0^2 (b_1 - b_3) + \alpha_1^2 (b_3 - b_0) + \alpha_3^2 (b_0 - b_1)]$$

$$U_3 = \frac{q_1 q_2}{2 \Delta} [\alpha_0^2 (b_2 - b_1) + \alpha_1^2 (b_0 - b_2) + \alpha_2^2 (b_1 - b_0)]$$

$$V_0 = \frac{q_1 q_2 q_3}{2 \Delta} [\alpha_1^2 (a_2 - a_3) + \alpha_2^2 (a_3 - a_1) + \alpha_3^2 (a_1 - a_2)]$$

$$V_1 = \frac{q_2 q_3}{2 \Delta} [\alpha_0^2 (a_3 - a_2) + \alpha_2^2 (a_0 - a_3) + \alpha_3^2 (a_2 - a_0)]$$

$$V_2 = \frac{q_1 q_3}{2 \Delta} [\alpha_0^2 (a_1 - a_3) + \alpha_1^2 (a_3 - a_0) + \alpha_3^2 (a_0 - a_1)]$$

$$V_3 = \frac{q_1 q_2}{2 \Delta} [\alpha_0^2 (a_2 - a_1) + \alpha_1^2 (a_0 - a_2) + \alpha_2^2 (a_1 - a_0)]$$

where

$$\Delta = q_1 q_2 q_3 [\alpha_1^2 (a_2 b_3 - a_3 b_2) + \alpha_2^2 (a_3 b_1 - a_1 b_3) + \alpha_3^2 (a_1 b_2 - a_2 b_1)].$$

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# Microstrip Loop Radiators for Medical Applications

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**Abstract**—Three microstrip loop radiators designed to operate at frequencies of 433, 915, and 1300 MHz are described. Empirical design methods and experimental results obtained with phantoms and human tissues are presented. The radiators are relatively well matched when applied to water boluses followed by muscle phantoms or human tissues. When used with the boluses, the radiators have circular surface-temperature distribution while the in-depth heating patterns are similar to those of the aperture-type radiators.

## I. INTRODUCTION

VARIOUS microstrip radiators and arrays of radiators for inducing local hyperthermia and for other medical applications of microwaves have been investigated [1]–[7].

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For example, an array of printed dipoles was developed to heat a large volume of tissue at 2450 MHz [1]. A coplanar-waveguide coupler was designed to minimize stray coupling in transmission measurements at 915 MHz [2]. Various microstrip-ring radiators were also constructed for inducing local hyperthermia at 915 and 2450 MHz [3], [5]. These radiators are matched when spaced a few millimeters from muscle or muscle phantom or when muscle is covered by a layer of fat. However, in these configurations, the heating pattern of the small fundamental-mode radiators is highly nonuniform because of the near-field effects. To improve the uniformity of the heating pattern, higher order mode, large-diameter radiators would be required. A microstrip slot radiator was also developed for inducing local hyperthermia as well as for medical diagnostics at 2450 MHz [4]. This radiator has relatively low leakage, is matched to human tissue, and has a heating pattern comparable to aperture-type radiators [8]. A microstrip rectangular patch antenna was found to be an efficient radiator when the width of the patch was one wavelength (or less) in the tissue [6].

Three microstrip loop radiators for medical applications